ARMOR POROSITY AND HYDRAULIC STABILITY OF MOUND BREAKWATERS

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Armor porosity significantly affects construction costs and hydraulic stability of mound breakwaters; however, most hydraulic stability formulas do not include armor porosity or packing density as an explicative variable. 2D hydraulic stability tests of conventional randomly-placed double-layer cube armors with different armor porosities are analyzed. The stability number showed a significant 1.2-power relationship with the packing density, similar to what has been found in the literature for other armor units; thus, the higher the porosity, the lower the hydraulic stability. To avoid uncontrolled model effects, the packing density should be routinely measured and reported in small-scale tests and monitored at prototype scale.

Keywords: mound breakwater; armor porosity; packing density; armor damage; armor unit; cubic block.

INTRODUCTION

When quarries are not able to provide stones of the adequate size and price, precast concrete armor units (CAUs) are required for the armor layer protecting large mound breakwaters. The first CAUs, introduced in the 19th century, were massive cubes and parallelepiped blocks with a very simple geometry. Since the invention of the Tetrapod in 1950, numerous precast CAUs with complex geometries have been invented to reduce the cost and to improve the armor layer performance.

The overall breakwater construction cost depends on a variety of design and logistic factors, like armor material (reinforced concrete, quality of unreinforced concrete, granite rock, sandstone rock, etc.), armor unit geometry (cube, Tetrapod, etc.), armor unit mass (3, 10, 40, 150-tonne, etc.), casting, handling and stacking equipment, transportation and placement equipment, energy, materials and personnel costs.

Assuming CAU structural integrity is guaranteed, using the appropriate unreinforced concrete or steel reinforced concrete (depending on CAU geometry and size), the hydraulic stability of an armor layer depends on four main factors:
1. Armor placement (random, patterned, ordered, specific, etc.).
2. CAU geometry (cube, Tetrapod, Dolos, Accropode, Xbloc, Cubipod, etc.).
3. Number of layers (n=1 or 2).
4. Armor porosity or packing density.

This paper focuses the attention on armor porosity, p, and the associated packing density, $\phi=n[1-p]$; special attention is given to conventional randomly-placed double-layer cube armors. The armor porosity or packing density directly affects concrete consumption and logistic costs. Hydraulic stability significantly decreases if packing density (dimensionless number of CAUs per unit surface) is lower than the recommended values. The literature regarding mound breakwaters protected with a variety of CAUs shows that hydraulic stability decreases if armor porosity increases; however, neither p nor ϕ is included in most commonly-used hydraulic stability formulas (e.g. Hudson's formula). This paper aims to explain the quantitative impact of packing density ($\phi=n[1-p]$) on the hydraulic stability of randomly-placed double-layer cube armors.

The stability coefficient (K_D) introduced by Hudson (1959) and popularized later by USACE (1984) has been used for decades to compare the hydraulic stability of different armor units in randomly-placed double-layer armors (with specifically recommended packing densities). Eq. 1 considering the equivalence $H=H_s$ is known as the generalized Hudson formula,

$$M = \frac{H_s^3 \rho_r}{K_D \left[\dfrac{\rho_r}{\rho_w}-1\right]^3 \cot \alpha} = \frac{H_s^3 \rho_r}{\Delta^3 K_D \cot \alpha} \qquad (1)$$

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where K_D is the stability coefficient, M is the armor unit mass, H_s is the significant wave height, α is the slope angle, $\Delta=(\rho_r/\rho_w-1)$, and ρ_r and ρ_w are armor unit and water mass densities, respectively. Eq. 1 can be re-written as a function of the stability number, $N_s=H_s/(\Delta D_n)=(K_D \cot\alpha)^{1/3}$, where $D_n=(M/\rho_r)^{1/3}$ is the equivalent cube size or nominal diameter. Eq. 1 does not take into account relevant environmental variables (wave period, storm duration, etc.) and structural variables (core permeability, armor porosity, etc.) which significantly affect the hydraulic stability of the armor layer; however, Eq. 1 is commonly used by practitioners to compare armor units in the preliminary design phase.

Over the last three decades, Level I (partial coefficients) and Level II and III probabilistic approaches have been proposed for use in the design of large breakwaters. Nevertheless, most projects and practitioners still refer to the stability coefficient (K_D) of the generalized Hudson formula ($H=H_s$), where p or ϕ is not included but assumed to be the recommended value. K_D was originally proposed by Hudson (1959) to characterize the hydraulic performance of conventional randomly-placed double-layer armors. Half a century later, it is still widely used to characterize a variety of CAUs placed on both single- and double-layer armors with different hydraulic performance. The stability coefficient concept is so popular among practitioners that K_D values are recommended also by patent owners of specifically-placed single-layer interlocking units, whose hydraulic performance does not follow the Hudson formula (interlocking units are usually less stable if they are placed on milder slopes). Medina et al. (2012) developed a methodology to calculate design K_{DS} and analyzed the implicit and explicit global safety factors associated with some recommended K_{DS} found in the literature. Medina et al. (2014) later highlighted the need to measure p and explicitly indicate the packing densities for small-scale models as well as prototype-scale structures, to effectively assess the impact of ϕ on hydraulic stability and safety factors associated with the K_{DS} used during the breakwater design process.

Using Eq. 1 for comparison, the higher the K_D, the lower the armor unit mass (lower concrete consumption, smaller handling and placement equipment and smaller filter stones). Since 1950, numerous armor unit geometries have been developed in the search for high values of K_D and the corresponding economic benefits. However, complex armor unit geometries with high values of K_D (e.g. Dolos) generate significant additional costs associated with complex formworks, high quality concrete and less efficient production and stacking procedures. Economic feasibility requires cost savings to be equal to or higher than the corresponding additional costs when compared to conventional double-layer cube armors. One of the key factors in assessing economic feasibility is the packing density, $\phi=n[1-p]$, which directly affects concrete consumption and logistic costs, as well as the hydraulic stability of the armor.

Since the invention of the Accropode^{TM} in 1980, several interlocking units have been invented for single-layer armoring, significantly reducing concrete consumption (see Vincent et al., 1989, and Holtzhausen and Zwamborn, 1991). Structural integrity and placement technique are critical issues to be addressed in order to guarantee adequate interlocking of units during service time (see Jensen, 2013). For single-layer armors, packing density and placement technique are usually explicitly prescribed by patent-owners; for randomly-placed double-layer armors, packing density is usually implicitly defined by engineering manuals (e.g. USACE, 1984, and CIRIA et al., 2007).

LITERATURE REVIEW

The porosity concept refers to the volume of voids in a granular system. Nevertheless, armor porosity, p, is not always easy to measure because armor thickness measurement is not straightforward. For CAUs orderly-placed on the slope, armor thickness is the distance between two clearly defined parallel surfaces; however, for randomly-placed CAUs, armor thickness has no clear definition. For randomly-placed units, armor thickness is usually referred to as n=1 (single-layer) or n=2 (double-layer) times the equivalent cube size, $nD_n=n(M/\rho_r)^{1/3}$. For each CAU, engineering manuals implicitly specify the packing density recommending a specific layer coefficient, k_Δ, associated with a specific nominal porosity, P, called "fictitious porosity" by Zwamborn (1978). The placing density ($\varphi[units/m^2]$) is indeed a real physical variable which is controlled by the placement grid, which may be related to either ϕ or the pair k_Δ and P, according to

$$\varphi = n(k_\Delta)(1-P)\left(\frac{\rho_r}{M}\right)^{2/3} = \frac{\phi}{D_n^2} = \frac{n(1-p)}{D_n^2} \qquad (2)$$

where ϕ is the packing density, p is the armor porosity, P is the nominal armor porosity, k_Δ is the layer coefficient, n is the number of layers, and $D_n=(M/\rho_r)^{1/3}$ is the equivalent cube size. Frens (2007) drew

attention to misinterpretations caused by the use of different criteria by different authors to establish the pair (k_Δ, P) for the same CAU because different pairs (k_Δ, P) can lead to the same placing density, (φ[units/m^2]). To prevent misunderstandings, this paper will refer to the number of layers, n, the packing density, $\phi=\varphi D_n^2$, and the armor porosity, p=1-ϕ/n. For instance, USACE (1984) and CIRIA (2007) recommend n=2, P=0.47 and k_Δ=1.10 for double-layer randomly-placed modified cube and cube respectively, which corresponds to n=2 and ϕ=1.17 (p=1-ϕ/2=41.5 %).

Armor placement and porosity can be easily controlled in small-scale tests (see Medina et al., 2010), because CAU models are placed by hand with perfect viewing conditions which do not exist in real breakwater construction. Prototype conditions involve the use of crawler cranes, wind, waves, poor underwater viewing and other restrictions that cause armor porosity changes (see Latham et al., 2013). Medina et al. (2014) provide a complete literature review revealing that a significant reduction in the ϕ below the recommended values results in a significant decrease in the hydraulic stability of the armor. Table 1 shows the main characteristics of the armor porosity related experiments found in literature.

Table 1. Main characteristics of the experiments given in the literature (from Medina et al, 2014).

Armor unit	n	Placement	First Author (year)	ϕ
	1	Random+Ordered	Hald (1998)	0.60
	1	Random+Ordered	Vandenbosch (2002)	0.70
	2	Random	USACE (1984)	1.26
Rock	2	Random	Latham (2002)	1.08≤ ϕ ≤1.29
	1	Random	Van der Meer (1999)	0.48
	1	Random	Vandenbosch (2002)	0.62, 0.73, 0.78
	1	Random	De Jong (2004)	0.62≤ ϕ ≤0.78
	2	Random	USACE (1984)	1.04
	2	Random	Van der Meer (1999)	0.88≤ ϕ ≤1.02
	2	Random	CIRIA (2007)	1.02
Tetrapod	2	Patterned	Gürer (2005)	0.80, 0.94
	1	Random	Van Gent (1999)	0.60, 0.70, 0.75
	1	Random	Medina (2010)	0.55≤ ϕ ≤0.65
	1	Regular	Van Buchem (2009)	0.65, 0.72 ,0.80
	1	Regular	Van Gent (2013)	0.60, 0.75
	2	Random	Vandenbosch (2002)	1.20, 1.40, 1.50
	2	Random	CIRIA (2007)	1.17
	2	Random	Gómez-Martín (2014)	1.26
Cube	2	Random	Medina (2014)	1.06, 1.17, 1.30
	2	Irregular	Yagci (2003)	0.94, 1.13
	2	Irregular	Yagci (2004)	1.07≤ ϕ ≤1.16
Antifer cube	2	Specific	Frens (2007)	0.90≤ ϕ ≤1.22
	2	Random	Carver (1978)	0.63≤ ϕ ≤0.83
	2	Random	Zwamborn (1978)	0.83≤ ϕ ≤1.15
	2	Random	USACE (1984)	0.83
Dolos	2	Random	Burcharth (1992)	0.61≤ ϕ ≤1.00
	1	Specific	Holtzhausen (1991)	0.64
	1	Specific	Burcharth (1998)	0.65
Accropode™	1	Specific	CLI (2012)	0.62≤ ϕ ≤0.65
	1	Random	Melby (1994)	0.54
Core-Loc™	1	Specific	CLI (2012)	0.62≤ ϕ ≤0.65
	1	Patterned	Bakker (2005)	0.55≤ ϕ ≤0.60
Xbloc^R	1	Patterned	DMC (2011)	0.58
	1	Random	Medina (2010)	0.49≤ ϕ ≤0.63
	1	Random	Gómez-Martín (2014)	0.60
	2	Random	Gómez-Martín (2014)	1.18, 1.20
Cubipod^R	2	Random	Pardo (2014)	1.09≤ ϕ ≤1.17

Experimental results given by different authors, corresponding to small-scale models protected by randomly-placed double-layer cube armors, 25%<p<40%, show cube armors with p=30% to be more stable than those with p=40%; however, CIRIA (2007) recommends p=41.5% for double-layer randomly-placed cube armors. Furthermore, using small-scale models and crawler cranes, Medina et al. (2010) reported that p<35% is not realistic when crawler cranes are used for placement and underwater viewing conditions are poor. The random placement by hand in perfect conditions observed in laboratory is easy and tends to reduce armor porosity below the recommended values. On the contrary, during real construction, wind, waves and poor viewing tend to increase armor porosity above recommended values. A higher p reduces the costs and concrete consumption but also reduces the armor hydraulic stability. Therefore, a scenario of small-scale models with lower-than-recommended p and real breakwaters built with higher-than recommended p values is likely occur, with a significant model effect that clearly reduces the design safety factors.

A similar trend was observed for randomly-placed double-layer Antifer cube armors (39%<p<55%). The stability number was higher for lower p; the observed increment of $N_s = H_s/[\Delta D_n]$ was much higher than that of $\phi = n[1-p]$. Small-scale experiments with other CAUs, such as Tetrapod, Dolos and Xbloc, also revealed this general trend both in single- and double-layer armors. Eq. 3 shows an explicit linear relationship between N_s and ϕ proposed by Burcharth and Liu (1992) when analyzing Dolos armors with slope H/V=cot α=1.5.

$$N_s = \frac{H_s}{\Delta D_n} = (47 - 72\,r)D^{1/3}N^{-0.1}\phi \tag{3}$$

where N_s is the stability number, r is the Dolos waist ratio, D is the relative number of displaced units, N is the number of waves and $\phi = n[1-p]$ is the packing density. Van der Meer (1999) also proposed explicit relationships between N_s and ϕ for randomly-placed single- and double-layer Tetrapod armors with cot α=1.5 and $0.48 \le \phi \le 1.02$; Eqs. 4 and 5 show the formulas proposed for surging and plunging waves, respectively

$$N_s = \frac{H_s}{\Delta D_n} = \left(3.75\frac{N_{od}^{0.5}}{N^{0.25}} + 0.85\left(0.40 + 0.61\left[\frac{\phi}{\phi_{SPM}}\right]\right)\right)s_{om}^{-0.1} \tag{4}$$

$$N_s = \frac{H_s}{\Delta D_n} = \left(8.6\frac{N_{od}^{0.5}}{N^{0.25}} + 3.94\left(0.40 + 0.61\left[\frac{\phi}{\phi_{SPM}}\right]\right)\right)s_{om}^{-0.1} \tag{5}$$

where N_s is the stability number; $s_{om} = H_s/[gT_m^2/2\pi]$ is the wave steepness, N is the number of waves, ϕ is the packing density, $\phi_{SPM} = 1.04$ is the recommended packing density for Tetrapod given by USACE (1984), and N_{od} is the relative armor damage. Without specific experimental support for cube CAUs, Van der Meer (1999) postulated Eq. 6 to take into account the influence of ϕ on the stability number of conventional randomly-placed double-layer cube armors

$$N_s = \frac{H_s}{\Delta D_n} = \left(6.7\frac{N_{od}^{0.4}}{N^{0.3}} + 1.0\left(0.40 + 0.61\left[\frac{\phi}{\phi_{SPM}}\right]\right)\right)s_{om}^{-0.1}\left(1 + 0.17\exp\left[-0.61\left(\frac{R_c}{D_n}\right)\right]\right) \tag{6}$$

where R_c is the crest freeboard, $D_n = (M/\rho_r)^{1/3}$ is the equivalent cube size, and the same notation as Eqs. 4 and 5 but using $\phi_{SPM} = 1.17$, which is the recommended packing density for modified cubes and cubes given by USACE (1984) and CIRIA (2007), respectively.

For a variety of CAUs, there is a considerable experimental evidence that packing density affects hydraulic stability; based on small-scale tests with different armor porosities, specific formulas have been proposed for Dolos and Tetrapod. The aim of this paper is to quantify the effect of $\phi = n[1-p]$ on hydraulic stability for conventional randomly-placed double-layer cube armors.

HYDRAULIC STABILITY TESTS

Results from small-scale hydraulic stability tests of two different randomly-placed double-layer cube armored breakwaters (H/V=2.0 and 1.5) were used to study the influence of armor porosity on hydraulic stability. The first model (H/V=2.0) corresponds to the secondary armor of the Punta Langosteira breakwater (A Coruña, Spain) and the second model (H/V=1.5) corresponds to the cube armored model described by Gómez-Martín and Medina (2014).

Punta Langosteira model (slope H/V=2.0)

Project CLIOMAR (2009-2011) involved overtopping and hydraulic stability testing of the double-layer 150-tonne cube armored Punta Langosteira breakwater (see Maciñeira et al., 2009). The experiments were carried out at the UPV wave flume (30.0x1.2x12 m) described by Medina et al. (2010) equipped with a piston-type wavemaker with AWACS active wave absorption. The secondary armor of Punta Langosteira breakwater was a conventional randomly-placed double-layer 15-tonne cube armor which protected the breakwater during the construction phase. The 1/46 small-scale hydraulic stability tests of the secondary armor at Low Water Level (LWL), with no or minor overtopping, were used to quantify the influence of armor porosity on the hydraulic stability of armor layers. Fig. 1 shows the 1/46 small-scale cross section. Water depth was h[m]=40 at prototype scale (h[cm]=87 in the wave flume). Runs of 1000 random waves of JONSWAP (γ=3.3) spectra were generated. Design significant wave height H_{sd}=12 cm (1/46 scale), corresponding to K_D=6.0 in Eq. 1, was used as reference for the hydraulic stability tests. The cube armor models were tested in non-breaking conditions; the dimensionless crest freeboards were R_c/H_{sd}=1.63 and R_c/D_n=4.8 and the water depth to design significant wave height ratio was h/H_{sd}=87/12=7.2.

In these experiments, the cube units did not have an exact cube geometry, but rather a slightly squared frustro-pyramidal geometry, similar to the conventional cubes commonly used at prototype scale to facilitate the vertical demolding. The characteristics of materials used in the small-scale model were:

    Cube units: M[g]= 156.2, ρ_r[g/cm³]=2.30 and D_n[cm]=4.08
    Toe berm rocks: M[g]= 50.0, ρ_r[g/cm³]=2.70 and D_n[cm]=2.65
    Filter layer rocks: M[g]= 15.3, ρ_r[g/cm³]=2.70 and D_n[cm]=1.78
    Core: M[g]=   0.8, ρ_r[g/cm³]=2.70 and D_n[cm]=0.67
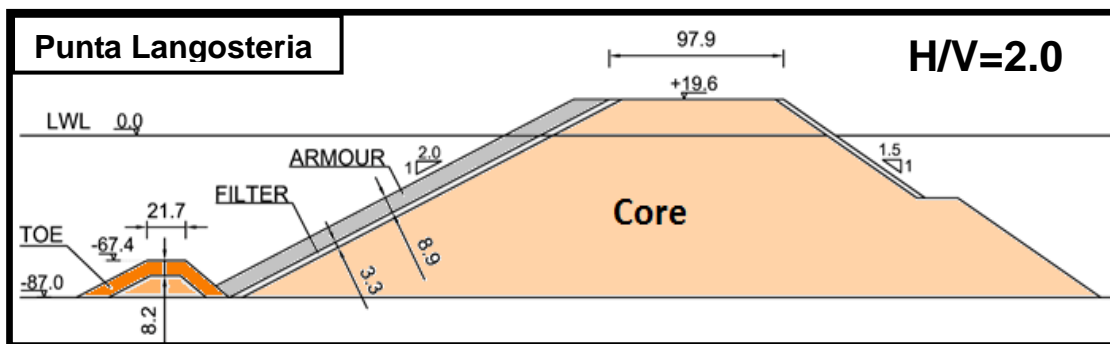

Figure 1. Punta Langosteira double-layer cube armored breakwater cross section during the construction phase (dimensions in cm).

Forty-eight small-scale tests were completed in eight test series. The test series were characterized by a model with an initial armor porosity (p≈37%, 41% and 47% corresponding to ϕ≈1.26, 1.18 and 1.06) and an approximately constant incident Iribarren number $3.0 \leq Ir_p = (1/2)/(2\pi H_s/gT_p^2)^{0.5} \leq 6.3$ or wave steepness (tan α=1/2 is constant in these experiments). After constructing each model, an orthogonal photograph of the armor was taken for reference, and runs of 50 regular waves of increasing energy were generated to settle the model. Once an initial armor movement was detected, irregular wave trains characterized by significant wave height and peak period, H_s and T_p, were generated. After each irregular wave run, armor damage was measured using the Virtual Net method (see Gómez-Martín et al., 2014), which take into account not only armor unit extraction but also armor unit movements within the armor layer (heterogeneous packing).

Significant wave height was increased progressively in steps, keeping approximately constant the wave steepness within the test series, e.g. H_s[cm]=10, 11, 12, 13, etc. Peak periods, T_p, were adjusted to keep constant the wave steepness and Iribarren number. Three low porosity cube armored models (p=37%) were tested for $Ir_p=(1/2)/(2\pi H_s/gT_p^2)^{0.5}$= 3, 4 and 5. Two medium porosity cube models (p=41%) were tested for Ir_p= 3 and 4. Finally, three high porosity cube models (p=47%) were tested for Ir_p =3, 5 and 6. Four capacity wave gauges were placed in the model area to measure wave elevation; the LASA-V method (see Figueres and Medina, 2004) was used to estimate incident and reflected waves.

The armor porosity and packing density, significant wave height and Iribarren's number given above were "a priori" target values; "a posteriori" values measured during the tests differed slightly from target values. Fig. 2 shows three cube armor models with armor porosities p=37%, 41% and 46% (ϕ=1.26, 1.18 and 1.08), which are above, near and below the recommended value (ϕ=1.17) given by USACE (1984) and CIRIA (2007).
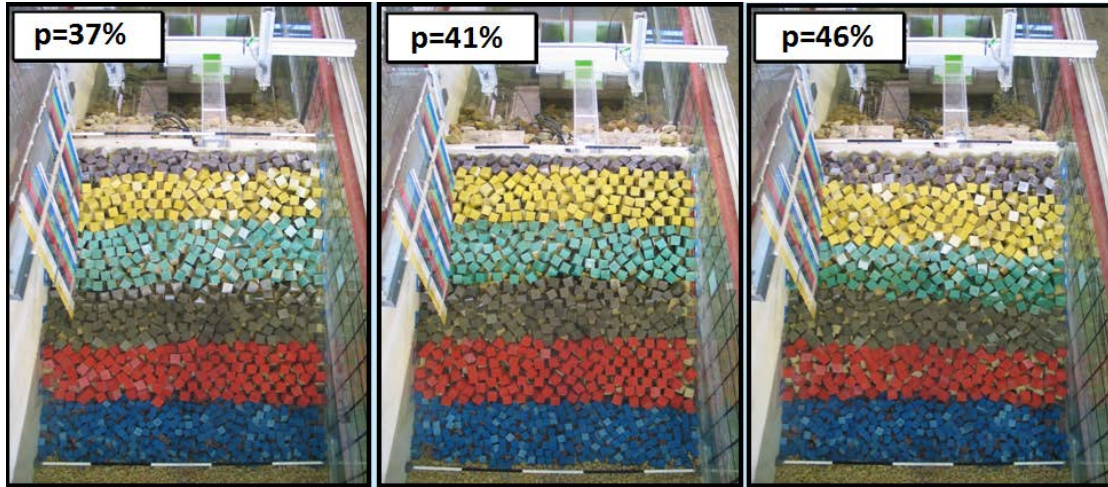

Figure 2. Double-layer cube armored breakwater models with different packing densities.

Slope H/V=1.5

Gómez-Martín and Medina (2014) used a methodology similar to that described above to study the hydraulic stability of randomly-placed double-layer cube armor with a slope H/V=1.5, steeper than the slope of the model of Punta Langosteira described previously. Fig. 3 shows the cross section of the breakwater model. Unlike the model in Fig. 1, structural differences include armor slope, the very high crest elevation and the lack of berm.
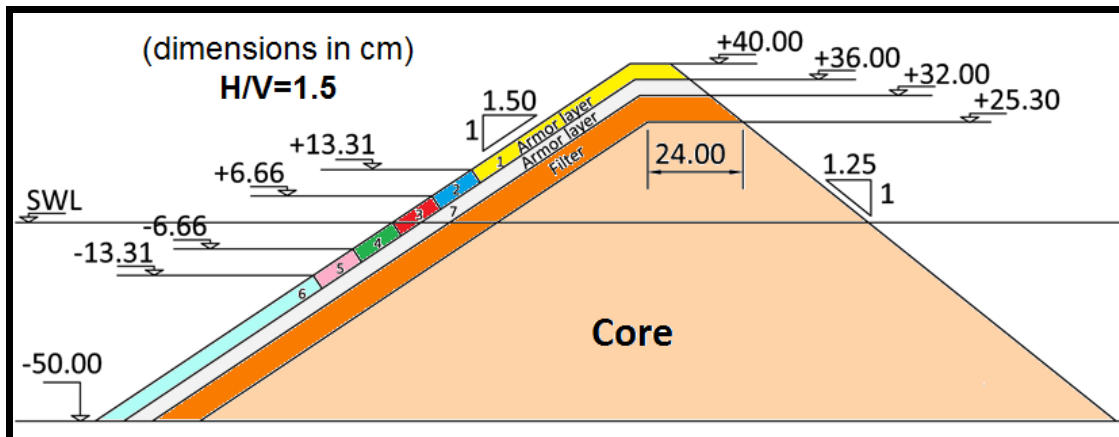

Figure 3. Double-layer cube armored breakwater cross section by Gómez-Martín and Medina (2014).

This model used regular cubes with M[g]=139.5, ρ_r[g/cm^3]=2.18 and D$_n$[cm]=4.00, slightly different from those used in the model described by Fig. 1. This model was also tested in non-breaking and non-overtopping conditions with a dimensionless crest freeboard R$_c$/D$_n$=10.0, and the water depth to design significant wave height ratio, h/H$_{sd}$=50/9.8=5.1.

ANALYSIS OF EXPERIMENTAL RESULTS

To analyze the influence of the packing density on the hydraulic stability of randomly-placed double-layer cube armors, Medina et al. (2014) proposed using the 4-parameter empirical formula given by Eq. 7 and a 5-parameter empirical formula based on Eq. 6 which gave slightly worse fitting

results. Therefore, the formula given by Eq. 7 was selected to describe the influence of packing density on the hydraulic stability of cube armors.

$$N_s = \frac{H_s}{\Delta D_n} = a_1 S_e^{a_2} \phi^{a_3} s_{op}^{a_4} \tag{7}$$

N_s is the stability number, S_e is the equivalent dimensionless armor damage using the Virtual Net method, $\phi = n(1-p)$ is the packing density at the initiation of each test series, $s_{op} = H_s/(gT_p^2/2\pi)$ is the wave steepness, and a_i (i=1 to 4) are four parameters to be estimated from the experimental data. The mean squared error (MSE) to variance ratio was used to measure the goodness of fit; the lower the MSE/Var, the better. If the number of tests is much higher than the number of parameters, MSE/Var is approximately the proportion of variance not explained by the formula. Taking the logarithms from both sides of Eq. 7, {a_1, a_2, a_3 and a_4} can be easily estimated by linear regression. Common linear regression software provides central estimations and standard deviation of parameters associated to the significant variables, which are used to determine the number of significant figures of each parameter.

Punta Langosteira model (slope H/V=2.0)

Taking the logarithms from both sides of Eq. 7 and following the methodology described above, the variables {S_e, ϕ and s_{op}} were found to be significant. The coefficient of variation of the parameters ranged from 13% to 20%; therefore, only parameters with two figures were considered and Medina et al. (2014) estimated the following empirical formula with a MSE/Var=23.5%

$$N_s = \frac{H_s}{\Delta D_n} = 0.74 S_e^{0.24} \phi^{1.6} s_{op}^{-0.15} \tag{8}$$

where cotα=2.0, $1.0 \leq S_e \leq 13.5$, $1.06 \leq \phi \leq 1.30$, $0.006 \leq s_{op} \leq 0.028$ ($3.0 \leq Ir_p \leq 6.3$). For a better assessment of the uncertainty associated with the use of Eq. 8, it is convenient to calculate the final prediction error (FPE) which takes into account not only MSE but also the number of test cases used for estimation and the parameters and the number of free parameters used in the formula. According to Barron (1984), FPE=MSE([48+4]/[48-4])=1.18xMSE. The 90% confidence interval associated with the estimations given by Eq. 8 is $N_s \pm 1.65(FPE)^{0.5} = N_s \pm 0.37$. Fig. 4 compares observed N_s and estimated N_s given by Eq. 8.
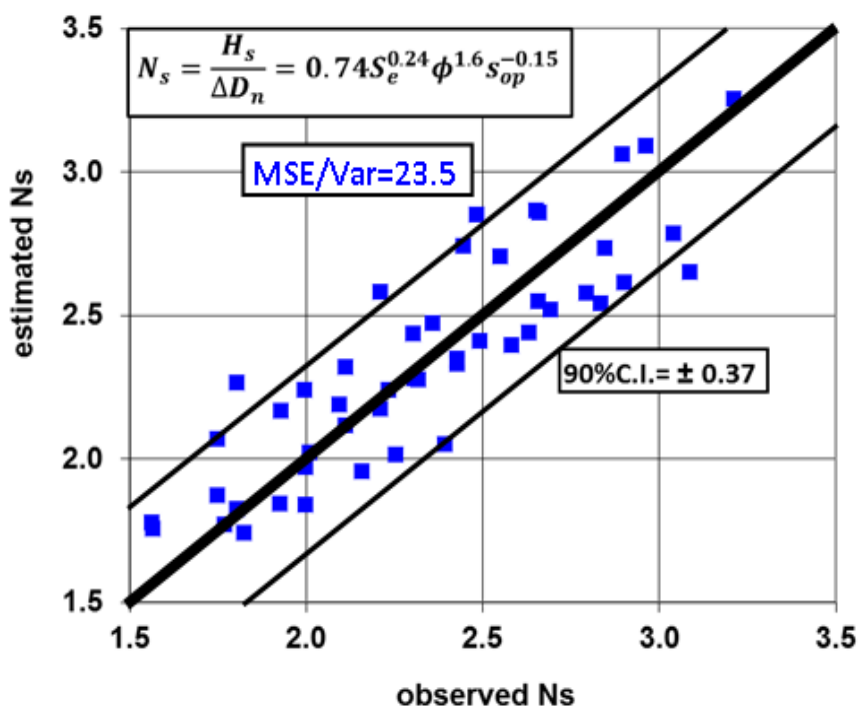

Figure 4. Comparison of observed and estimated stability numbers using Eq. 8.

Taking into consideration only the 48 tests carried out for the H/V=2.0 model of Punta Langosteira breakwater described in Fig. 1, a 1.6-power relationship was found between the stability number Ns and the packing density ϕ. With a level of significance of 5%, $1.2 < a_3=1.6 < 2.0$. The results of these experimental tests strongly support a relevant influence of the armor porosity on the hydraulic stability of the armor layers.

Slope H/V=1.5 and H/V=2.0

Using the data from Gómez-Martín and Medina (2014) corresponding to a H/V=1.5 breakwater model, described in Fig. 3, in the same range of variables ($1.0 \leq S_e \leq 13.5$, $1.06 \leq \phi \leq 1.30$, $3.0 \leq Ir_p \leq 6.3$) a new generalized formula is obtained with a broader experimental support ($1.5 \leq H/V \leq 2.0$).

In addition to the 48 hydraulic stability tests corresponding to the eight models with H/V=2.0 previously described, 66 hydraulic stability tests corresponding to nine models with H/V=1.5 were used to obtain the new formula. Using the same methodology described before, Medina et al. (2014) added to wave steepness, s_{op}, the Iribarren number, Ir_p, and the armor slope, cot α. The linear regression rejected the variables Ir_p and s_{op} with a level of significance of 5%; only armor damage (S_e), packing density (ϕ) and slope (cot α) were found to be significant explicative variables. The coefficient of variation of the parameters ranged from 8% to 40%; therefore, only parameters with two figures were considered and Medina et al. (2014) proposed the following empirical formula with a MSE/Var=29.8%

$$N_s = \frac{H_s}{\Delta D_n} = 1.31 S_e^{0.21} \phi^{1.2} (\cot \alpha)^{0.20} \tag{9}$$

where $1.5 \leq \cot \alpha \leq 2.0$, $1.0 \leq S_e \leq 13.5$ and $1.06 \leq \phi \leq 1.30$. FPE=MSE([114+4]/[114-4])=1.07xMSE. The 90% confidence interval associated with the estimations given by Eq. 9 is $N_s \pm 1.65(FPE)^{0.5} = N_s \pm 0.39$. Fig. 5 compares observed N_s and estimated N_s given by Eq. 9.
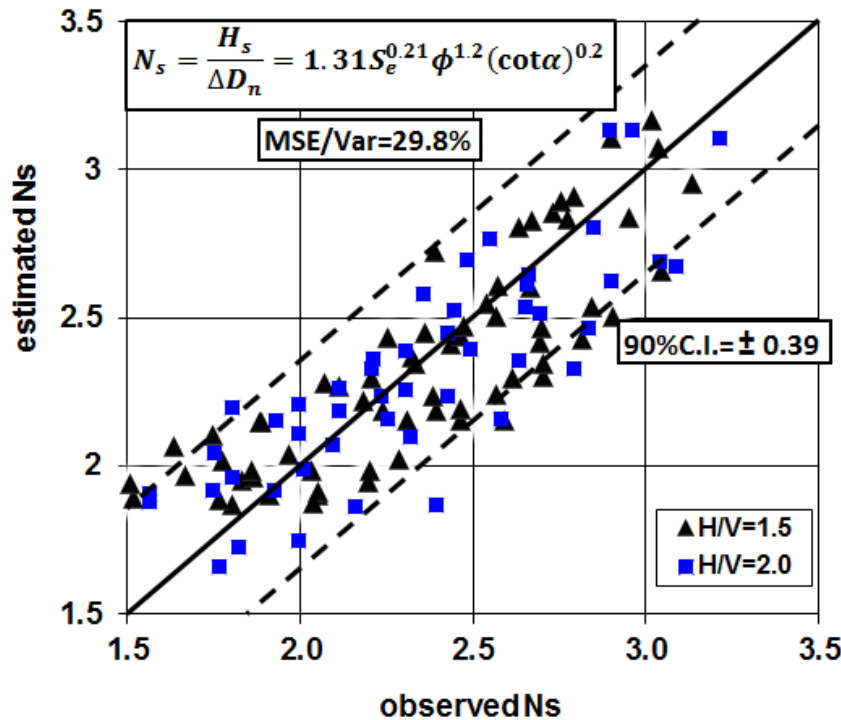

Figure 5. Comparison of observed and estimated stability numbers using Eq. 9.

The generalized Hudson formula, Eq. 1 or $N_s = H_s/(\Delta D_n) = (K_D \cot\alpha)^{1/3}$, has qualitative and quantitative resemblances to Eq. 9, taking into consideration the recommended packing density ϕ=1.17 given by USACE (1984) and CIRIA (2007) and the design dimensionless armor damage. The 0.21-

power relationship between stability number N_s and equivalent dimensionless armor damage S_e for double-layer cube armors is similar to the 0.20-power relationship for double-layer rock armors given by Van der Meer (1988) and also by Medina et al. (1994) analyzing data from USACE (1984). The 0.20-power relationship between stability number N_s and cot α for double-layer cube armors is not very different from the 0.33-power relationship given by the generalized Hudson formula. The 1.2-power relationship between stability number N_s and packing density φ for double-layer cube armors is similar to the 1.0-power relationship found by Burcharth and Liu (1992) for double-layer Dolos armors; a positive correlation between packing density and hydraulic stability of the armor layer is relevant for cubes, Dolos and many other armor units found in the literature.

Concrete consumption in the armor layer

Construction site conditions (water depth, available materials, etc.) and design storm (H_{sd}, T_p, wave direction) are given in the preliminary design phase of any mound breakwater. For double-layer cube armors, the concrete consumption required for an armor layer is proportional to 1-p=φ/2; the volume of the armor layer is approximately proportional to the breakwater length, the armor layer thickness and the armor width. For double-layer armors, armor thickness is $2D_n$ and armor width is approximately proportional to H_{sd}/sin α (non-breaking conditions) or h/sin α (breaking conditions). Fig. 6 shows a sketch of a typical double-layer cube cross section in breaking conditions; armor thickness is $2D_n$ and armor width is approximately proportional to (h+Δh+R_c)/sin α, where h is the water depth, Δh is the tidal range and Rc is the armor crest elevation. Assuming Rc is directly proportional to h (Δh<<h), armor width is approximately proportional to h/sin α, where h is given by construction site conditions. Therefore, for double-layer cube armors, the concrete consumption is approximately proportional to (φ/2)2D_n(1/sin α)=φD_n/sin α.
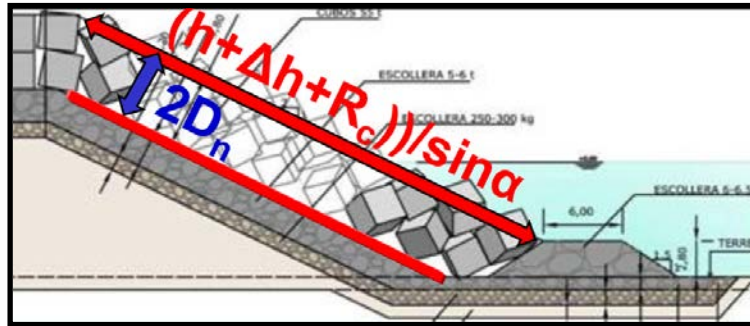

Figure 6. Sketch of a typical double-layer cube armor in breaking conditions.

Considering Eq. 9, in the preliminary design phase, concrete consumption of randomly-placed double-layer cube armors is approximately proportional to

$$\frac{\phi D_n}{\sin \alpha} = \frac{k}{\phi^{0.2} \sin \alpha (\cot \alpha)^{0.2}} \qquad (10)$$

where φ is the packing density (1.06≤φ≤1.30), D_n=(M/$ρ_r$)$^{1/3}$ is the equivalent cube size or nominal diameter, α is the slope angle (1.5≤cot α≤2.0), and k is a constant depending on construction site and design conditions. According to Eq. 9, packing density φ has a relevant impact on hydraulic stability; if φ is correctly controlled at prototype scale, Eq. 10 shows that double-layer cube armors with a higher packing density (lower armor porosity) requires less concrete consumption.

Comparing a high porosity cube armor (p=46%) with a low porosity cube armor (p=37%) in equal conditions, the high porosity armor require armor units 74% heavier than the low porosity armor (Eq. 9), and 3% more concrete consumption (Eq. 10). From the design point of view, low porosity cube armors are preferable than high porosity armors; however, cube units may be difficult or impossible to place correctly if armor porosity is too low, depending on placement techniques and environmental conditions at the construction site (see Pardo et al., 2014).

Comparing armors with steeper and milder slopes, a steeper cube armor having H/V=1.5 requires armor units 19% heavier than the armor having H/V=2.0 (Eq. 9), and 15% less concrete (Eq. 10). Because of the significant reduction in concrete consumption, considering Eqs. 1 or 9 and Eq. 10, steeper armors having H/V=1.5 are usually preferable to milder armors having H/V=2.0.

CONCLUSIONS

In order to quantitatively study the influence of armor porosity on the hydraulic stability of randomly-placed double-layer cube armored breakwaters, experimental results are analyzed from 48 hydraulic stability tests corresponding to eight H/V=2.0 cube armored models and 66 tests corresponding to nine H/V=1.5 cube models, described in detail by Medina et al. (2014). A similar experimental methodology and the same type of analysis was applied to all 114 hydraulic stability tests of permeable core, non-breaking and non-overtopping conditions having slopes H/V=1.5 without toe berm and H/V=2.0 with a large toe berm.

Stability number, N_s, is dependent on: (1) the equivalent dimensionless armor damage, S_e, (2) the packing density, ϕ, and (3) the slope, cot α. Wave steepness and Iribarren's number were rejected with a level of significance of 5%. For double-layer randomly-placed cube armors with slopes 1.5≤H/V≤2.0, N_s can be estimated using Eq. 9 with a 90% confidence interval Ns±0.39 and variables in the ranges 1.0≤S_e≤13.5 and 1.06≤ϕ≤1.30. A relevant 1.2-power relationship was found between stability number, N_s, and packing density, ϕ; it is proved that armor porosity has a significant impact on hydraulic stability. If the packing density of a small-scale model is different from the packing density of the corresponding prototype, the model effect may be relevant. The risk of armor failure increases significantly if the packing density at prototype scale is reduced significantly below the design or tested value.

Taking into account that concrete consumption is approximately proportional to $\phi D_n/\sin$ α, Eq. 10 can be used to estimate relative concrete consumption of randomly-placed double-layer cube armors in the preliminary design phase. Concrete consumption increases if slope angle and packing density decreases favoring steeper slopes H/V=1.5 and low porosity armors. Nevertheless, high porosity armors are usually easier to build and the placement of cubic units with poor underwater viewing conditions is a challenge at p<35%.

The 1.2-power relationship found for randomly-placed double-layer cube armors may be different for other CAUs and placement techniques (patterned, specific, etc.). For instance, orderly-placed single-layer cube armors have different construction challenges and hydraulic responses (see Van Gent and Luis, 2013). Furthermore, depth-limiting conditions, core permeability, toe berm, crown wall, as well as other structural and environmental factors, may significantly modify the performance of the armor layer. Nonetheless, there is clear experimental evidence of a strong influence of armor porosity on hydraulic stability, found in the 114 tests corresponding to 17 double-layer cube armored breakwater models analyzed in this study, as well as in many other experiments described in the literature and corresponding to a variety of CAUs.

Armor porosity and packing density should be taken seriously as a key factor during the design process, in the small-scale tests and during construction. To avoid undesired model effects, packing density should be measured and reported in small-scale armor hydraulic stability tests and routinely controlled and monitored during the construction phase.

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